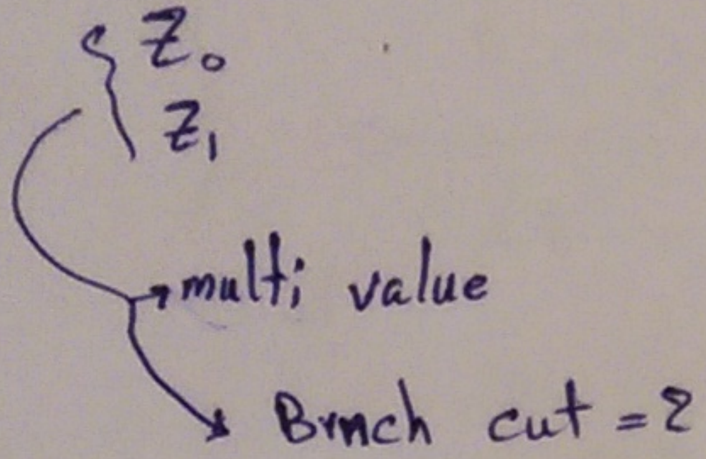
$$z^{1/2} = r^{1/2} e^{i\theta/2}$$

$$= r^{1/2}\left[\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right]$$



$\cos\theta \longrightarrow 2\pi$

$\theta \longrightarrow 2\pi$

$\frac{\theta}{2} \longrightarrow \pi$

application of complex integration.

$$I = \int_0^{2\pi} F(\sin\theta, \cos\theta)\, d\theta$$

$$\sin\theta = \frac{1}{2i}\left(z - \frac{1}{z}\right)$$

$$\cos\theta = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

$$d\theta = \frac{dz}{iz}$$

$$I = \int_0^{2\pi} F(\sin\theta, \cos\theta)\, d\theta$$

$$\sin n\theta = \frac{1}{2i}\left(z^n - \frac{1}{z^n}\right)$$

$$\cos n\theta = \frac{1}{2}\left(z^n + \frac{1}{z^n}\right)$$

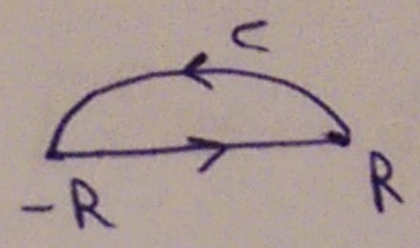
$$d\theta = \frac{dz}{iz}$$

$$\int_0^{2\pi} = 2\int_0^{\pi} \quad , \quad \int_{-2\pi}^{2\pi} = 2\int_0^{2\pi}$$

$\oint_{|z|=1}$

[b]

$$I = \int_{-\infty}^{\infty} = \oint F(z)\,dz$$



$$e^{iaz} = \cos az + i \sin az$$

$$\tfrac{1}{2}\int_{-\infty}^{\infty} = \int_{0}^{\infty}$$

---

evaluate

$$\int_0^{2\pi} \frac{d\theta}{2+\sin\theta} = \oint \frac{\frac{dz}{iz}}{2 + \frac{1}{2i}\left(z - \frac{1}{z}\right)}$$

$$= \int_0^{2\pi} \frac{dz}{iz\,2z + \frac{1}{2}(z^2-1)} = \int_0^{2\pi} \frac{dz}{\frac{1}{2}z^2 + iz\,2z - \frac{1}{2}}$$

$$= \oint \frac{dz}{(z-(\sqrt{3}-2)i)\,(z-(-2-\sqrt{3})i)}$$

$$\oint_{|z|=1} \frac{\frac{1}{z-(-2-\sqrt{3})i}}{z-(\sqrt{3}-2)i} = 2\pi i \frac{1}{2\sqrt{3}\,i}$$

$$\int_0^{2\pi} \frac{\cos 3\theta}{1-\cos\theta}\, d\theta = \int_0^{2\pi} \frac{\frac{1}{2}\left(z^3+\frac{1}{z^3}\right)}{1-\frac{1}{2}\left(z+\frac{1}{z}\right)} \frac{dz}{iz}$$

$$= \oint_{|z|=1} \frac{\frac{1}{2}\,\frac{z^6+1}{z^3}}{iz - \frac{i}{2}z^2 + i}\, dz$$

$$= \oint_{|z|=1} \frac{z^6+1}{z^3\left(-iz^2+2iz+2i\right)}\, dz$$

$$= i\oint_{|z|=1} \frac{z^6+1}{z^3(z-1)^2}$$

$$= i\left[\int \frac{\frac{z^6+1}{(z-1)^2}}{z^3} + \oint \frac{\frac{z^6+1}{z^2}}{(z-1)^2}\right.$$

$$= i\left[\frac{2\pi i}{n!} * (A) + \frac{2\pi i}{n!}(B)\right]$$

[d]

$$\int_0^\infty \frac{\cos ax}{x^2+1}\,dx = \tfrac{1}{2}\int_{-\infty}^{\infty} \frac{e^{iaz}}{z^2+1}\,dz$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{e^{iaz}}{(z-i)(z+i)}\,dz$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} \frac{\dfrac{e^{iaz}}{z+i}}{z+i}\,dz$$

$$= \tfrac{1}{2} * 2\pi i * \text{-------} = \frac{e^a}{-2i} * \tfrac{1}{2}$$

$$= \frac{-\pi}{2} e^a .$$